Expected Time to Rendezvous in Multi-hop Cognitive Radio Networks

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***Abstract*—The design of blind rendezvous strategies for dynamic spectrum access in multi-users, multi-hop networks is a challenging task. In this paper, a two radios synchronization strategy that minimizes the time to rendezvous in multi-users networks is analyze. In fact, it has been recently demonstrated that this synchronization strategy provides optimum results in terms of maximum time to rendezvous (MTTR). In this paper, the analysis to the expected time to rendezvous (ETTR) and derive analytical expressions for its numerical computation is extended. The estimated ETTR is compared with those obtained by simulation, using a representative blind rendezvous channel hopping sequence. The results indicate that the numerical expressions provide good estimates of the actual ETTR. Moreover, the ETTR achieved by the proposed synchronization strategy has proven to be bounded by the MTTR of a pair-wise rendezvous process irrespective of the number of users and the network topology. This property makes the analyzed rendezvous mechanism, and particularly the synchronization strategy, suitable for the deployment of large and dense networks.**

***Index Terms*—Blind rendezvous, channel hopping sequences, cognitive radio networks, multi-hop networks.**

I. Introduction

Cognitive radio networks (CRNs) has emerged as a technology that can solve the shortage of the spectrum by opportunistically exploiting it. To establish a communication link, two cognitive users, also known as Secondary Users (SUs), should meet on a common channel and exchange handshake information. This process is referred to as rendezvous and has drawn the attention of many researchers in recent years.

Numerous works have been published proposing different rendezvous algorithms, some of them [1]–[4] consider the presence of a dedicated common control channel (DCCC) in which all secondary users match and exchange the signaling information. However, the use of a DCCC is prone to congestion, especially for large networks, and represents a single point of failure. An alternative approach, to overcome this problems, is to use channel hopping (CH) sequences, in which each SU visits the network channels following a predefined strategy in order to achieve rendezvous in any of the available channels. This process is referred to as blind rendezvous [5]–[11]. This paper focuses on the blind rendezvous strategies in multi-hop ad-hoc networks.

The most important metric to assess the performance of such algorithms is the time, measured in time slots, needed for rendezvous, which is commonly referred as the time to rendezvous (TTR). The TTR has been evaluated in the majority of the works from its expected value (ETTR) and its maximum value (MTTR) in scenarios containing exactly two SUs (i.e. a pair-wise rendezvous). A simple variant to extended this process to a multi-user, multi-hop network was presented by Liu et al. in [5], [7]. More elaborated strategies, specially regarding the synchronization process, were proposed in [12].

When a pair-wise rendezvous occurs in a multi-hop network and the CH sequences of the involved SUs are synchronized, the TTR for these SUs with the others cognitive users can be affected. In what follows, the modification of the link's TTR after a pair-wise rendezvous is refer as a TTR renewal process. The CH synchronization strategies proposed in [12] attempt to avoid that after a pair-wise rendezvous the TTR of the remaining links will be modified (i.e. prevents TTR renewal to occur). This has proven to minimize the network MTTR, and accordingly a reduction of the ETTR is expected. One way to avoid the TTR renewal process, as proposed in [12], is by using multiples radios, such as when a pair-wise rendezvous occurs, one of the radios is synchronized while the other continues its current jumping sequence. The use of multiple radios has also been considered in [11], [13], [14] with the aim of reduce the TTR, but no special attention have been put in the synchronization strategies in order to avoid TTR renewals.

This paper extends the analysis made in [12] for the estimation of the ETTR in multi-hop ad-hoc networks. A two radios synchronization strategy is employed to avoid TTR renewals. Analytical expression for ETTR computation is derived under tight assumptions. Also, the effect of the presence of Primary Users (PUs; i.e. the licensed users which have priority access over the communication channels), is considered. The estimated ETTR is compared with those obtained by computational simulation using a representative blind rendezvous CH sequence. The results indicate that the analytical expressions provide good estimation of the actual ETTR. Moreover, the ETTR achieved by the proposed synchronization strategy has proven to be bound by the MTTR of a pair-wise rendezvous process irrespective of the number of SUs and the network topology. This property makes the analyzed rendezvous mechanism and the proposed synchronization strategy suitable for the deployment of large and dense networks.

II. System Model

A CRN consisting of M, $M \geq 2$ users, who coexist with one or more PUs in the same geographical area. The PUs are the owners of the licensed spectrum, which can be divided into N, $N \geq 2$ non-overlapping channels. These channels are indexed uniquely as $1, 2, ...N$. The time is divided into slots of equal duration. Each SU is equipped with two radios. Every time two or more network devices achieve rendezvous, their CH sequences are synchronized in one of its radios (always the same radio) and start to follow the same jump pattern, while the other radio continues its current jumping sequences. In that way, for the radio that keeps its current CH sequences the TTR for each link remains unchanged and no TTR renewal is produced. Figure 1 depicts the synchronization process in a simple network scenario with four SUs.

The network topology is assumed to be general. In Figure 1, a linear network topology is used for easy of explanation. The SUs are represented with circles, and the inner circles represent the radios, which begin to jump with the same CH sequence. However, since the SUs initiated the rendezvous process at different times, its CH sequences are shifted. After a pair-wise rendezvous, the upper radios are synchronized while the bottom radios maintain its current sequence. The links are indicated by the straight lines that connect them. The numbers on the links are the remaining TTR for each pair-wise rendezvous. Once a pair-wise rendezvous occurs the remaining TTR of the link is set to zero. The network rendezvous is achieved when all the remaining TTR are zero.

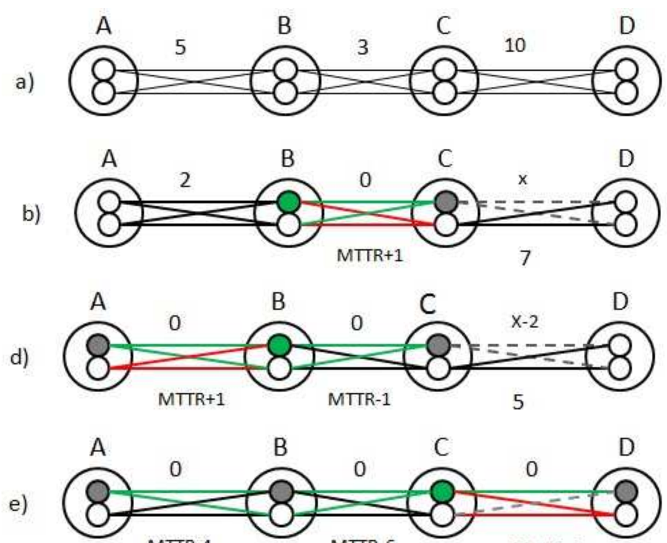

Fig. 1: Example of parallel multi-hop rendezvous model network.

In Figure 1 a) the SUs B and C are the first to achieve pair-wise rendezvous. After 3 time slots, SU B and C met in a common channel. For the upper radios, the green circle represents the radio that maintains its current CH sequence and the gray circle represents the radio that updates its CH sequence in a pair-wise rendezvous. When two SUs are in the same channel, the links of the radios that begin jump together are represented in green (TTR $= 0$), the links of the radios that continues its current sequences are marked in red (here the worst case scenario in which the shifted sequences have just one interception point is considered, hence after rendezvous if they are not synchronized the sequences agree again in the same channel at TTR $=$ MTTR $+ 1$) and the links between the radio that renew its sequence and the radios of the SU neighbor with discontinuous gray lines (TTR $= X; 0 \leqslant X \leqslant$ MTTR). Figure 1 b) depicts what happens after the first pair-wise rendezvous. In this example, the upper radio of SU C starts to follow the CH sequences of the upper radio of SU B, and the bottom radios of each user continue with their current CH sequences. Since the upper radio of SU C has adopted the CH sequence of SU B, the remaining TTR between upper radio of SU C and the two radio of the SU D has been reset, but TTR between bottom radio of SUs C and the two radios of SU D is maintained. The network (from the rendezvous point of view) can be equally modeled by a network with $M-1 = 3$ SUs, where the upper radio of SUs B and C jump together. Two time slots latter, the upper radio of SU A will join to upper radios of SUs B and C, and the bottom radios of SU A and SU B continue with their current CH sequences; this is represented in Figure 1 c). Five time slots latter, the upper radio of SU D and bottom radio of SU C met in a common channel and all the upper radios are synchronized. In this case, the TTR for the whole network is $3 + 2 + 5 = 10$.

The reasoning in the example of Figure 1 is valid for any distribution of nodes in the network, so the analysis in the following sections applies regardless of network topology. Also it is worth to note that the use of two radios in each node ensures that the synchronization strategy avoids TTR renewals as concequence, the resulting TTR of the network will always be equal to the largest of the TTRs; which is bounded by the MTTR for a pair-wise rendezvous as discussed in [12].

Another variant to be considered, with the synchronization procedure described above, is to start with shifted sequences in the two radios of each SU. Thus there are four different values of TTR for each pair of radios combination in a link. For a given rendezvous algorithm, the shift between the sequences can be pick such that links TTR is significantly reduced. As a result, a reduction of the network TTR is expected. This will be further elaborated in Section IV for the rendezvous algorithm presented in [10].

III. ETTR Calculation

In this section, an ad-hoc network with M cognitive users and a general rendezvous algorithm is considered. The ETTR and MTTR of the network, composed by M SUs, will be denoted ETTR_M and MTTR_M respectively. Following this notation, the MTTR for a pair-wise rendezvous will be written as MTTR_2. The ETTR_M is calculated under the following assumptions: (*i*) the TTR of the links are modeled as *i.i.d* variables with a uniform discrete distribution on the interval

$[0, \mathrm{MTTR}_2]$, and (*ii*) the CH synchronization strategy guarantees that no TTR renewal will occur during the rendezvous process (e.g. by using multiples radios as described in Section II).

Lets X_i be the TTR of the i^{th} link. Then, in virtue of (*ii*), the TTR for the whole network (TTR_M) can be obtained as:

$$\mathrm{TTR}_M = \max\big\{X_i\big\}. \tag{1}$$

In this case, given that $X_i \leq \mathrm{MTTR}_2$ for all i:

$$\mathrm{MTTR}_M = \max\big\{\mathrm{TTR}_M\big\} = \mathrm{MTTR}_2. \tag{2}$$

The fact that the MTTR_M is equal to the MTTR of a pair-wise rendezvous is one of the strongest and most promising characteristic of the synchronization strategies in which the TTR renewal is avoided. A direct implication of this is that it can be established an upper bound for the ETTR_M as:

$$\mathrm{ETTR}_M = \mathbf{E}\big\{\mathrm{TTR}_M\big\} \leq \mathrm{MTTR}_2 \tag{3}$$

Since TTR_M is a non negative integer random variable, on the interval $[0, \mathrm{MTTR}_2]$, the exact value of the ETTR_M can be calculated as:

$$\mathrm{ETTR}_M = \sum_{x=0}^{\mathrm{MTTR}_2} \big(1 - P(\mathrm{TTR}_M \leq x)\big), \tag{4}$$

Given assumption (*i*) and noticing that $M-1$ pair-wise rendezvous have to occur before network rendezvous can be achieved, the CDF of TTR_M can be expressed as follows:

$$\begin{aligned} P(\mathrm{TTR}_M \leq x) &= \prod_{i=1}^{M-1} P(X_i \leq x) = \prod_{i=1}^{M-1} \frac{x+1}{\mathrm{MTTR}_2 + 1} \\ &= \left(\frac{x+1}{\mathrm{MTTR}_2 + 1}\right)^{M-1}, \end{aligned} \tag{5}$$

then by substituting (5) in (4) is obtained:

$$\mathrm{ETTR}_M = \sum_{x=0}^{\mathrm{MTTR}_2} \left(1 - \left(\frac{x+1}{\mathrm{MTTR}_2 + 1}\right)^{M-1}\right) \tag{6}$$

It's interesting to note that as the number of SUs increases, the ETTR_M asymptotic tends to MTTR_2. When M tends to infinity, (6) can be written as:

$$\begin{aligned} \mathrm{ETTR}_\infty &= \lim_{M\to\infty} \sum_{x=0}^{\mathrm{MTTR}_2} \left(1 - \left(\frac{x+1}{\mathrm{MTTR}_2 + 1}\right)^{M-1}\right) \\ &= \mathrm{MTTR}_2. \end{aligned} \tag{7}$$

This result is consistent with (3). Moreover, it holds for the general case, irrespective of the TTR distribution, as long as the links TTR continue to be *i.i.d*; which is, indeed, the worst case scenario.

So far, we haven't analyzed the influence of PUs in the ETTR. Lets model the presence of the PUs as a bernoulli process, where the probability that a channel is being used by the licensed user in any given time slot is equal to the PU's channel utilization ρ. In this case equation (5) modifies as:

$$P(\mathrm{TTR}_M \leq x) = \prod_{i=1}^{M-1} \sum_{j=0}^{x} \frac{(1-\rho)\rho^{\left\lfloor \frac{j}{\mathrm{MTTR}_2+1} \right\rfloor}}{\mathrm{MTTR}_2 + 1}, \tag{8}$$

where the $\lfloor \cdot \rfloor$ operator rounds the argument to the nearest integer towards zero. Accordingly the ETTR_M can be computed as:

$$\mathrm{ETTR}_M = \sum_{x=0}^{\infty} \left(1 - \left(\sum_{j=0}^{x} \frac{(1-\rho)\rho^{\left\lfloor \frac{j}{\mathrm{MTTR}_2+1} \right\rfloor}}{\mathrm{MTTR}_2 + 1}\right)^{M-1}\right). \tag{9}$$

Its worth to note that for $\rho = 0$ (i.e. no PUs activity) equations (8) and (9) reduce to (5) and (7) respectively.

Since the argument of the external summation in equation (9) rapidly tends to zero as x increases, we can get a good approximation of the ETTR_M with a relatively small set of summation terms that can be easily computationally implemented.

IV. Numeric results

We use computer simulations to evaluate the solutions obtained by the proposed numerical method. Simulations were implemented in a discrete event simulator. To estimate the network ETTR, $5\mathrm{x}10^6$ independent simulations were performed, randomly distributing the initial TTR for the links.

The first experiment follows the model assumptions made in Section III and in what follows will be referred as a general model. The MTTR_2 is set to 30 time slots and the numbers of SUs is varied from 2 to 45. The results are shown in Figure 2 for a multi-hop ad-hoc network without PUs interference. It is assumed that each SU is equipped with two radios that begin to jump together; when two SUs meet in a common channel rendezvous occurs and CH sequences are synchronized in one of its radios while the other radio continues its current jumping sequences as explained in Section II.

Additionally, Figure 2 includes the results for analytical model by using equation (6). The comparison between general and analytical models validates the analysis presented in the previous section.

To evaluate how the synchronization strategy behaves with a real rendezvous sequence, it was implemented with the Short Sequence Based (SSB) rendezvous algorithm [12]. This sequence has the lower MTTR of all the CH sequences known so far. Simulations were carried out using N = 16 common available channels; leading, in this case, to a pair-wise $\mathrm{MTTR}_2 = 2(N-1) = 30$ time slot.

For all curves shown in Figure 2, the ETTR_M increases with M and asymptotically tends to MTTR_2 as predicted by expression (7). For the SSB algorithm with $M = 2$ SU the

$\mathrm{ETTR}_2 = 14.4728$ which exactly matches the numerical result obtained in [10]. The ETTR_M begins to grow in a similar way to the general model, decreasing the gap between the curves as M increases, since the upper bound, MTTR_2, is the same in both cases.

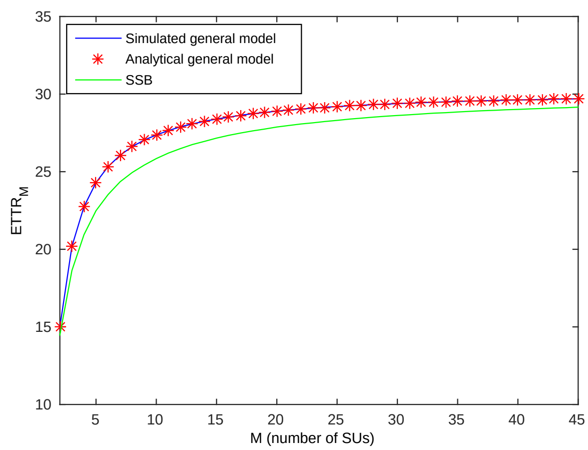

Fig. 2: ETTR for multi-hop ad-hoc network, simulated and computed values for the general model and SSB with two radios, by varying the number of SUs.

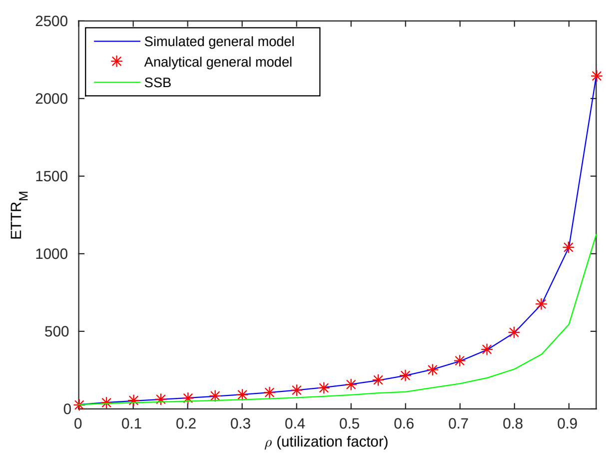

Fig. 3: ETTR for multi-hop ad-hoc network with M=20 SUs, simulated and computed values for the general model and SSB with two radios, by varying ρ.

In the second experiment, the PUs activity is considered randomly distributed over all channels with utilization factor ρ, it varies from 0 to 0.95 and use $M = 20$ SUs for simulations. The results are shown in Figure 3 for a multi-hop ad-hoc network with PUs interference. The simulation results are compared with those obtained by the proposed mathematical method and perfect match is observed for the general model.

When comparing with SSB, there is a difference that increases with ρ, between the computed ETTR_M for the general model and the simulation that uses real CH sequence. The reason for this behavior is that the assumptions made for the general model do not apply straightforwardly to SSB. For instance, assumption (i) represents the worst case scenario, while in the SSB algorithm the TTR of the links are not uniform distributed ($\mathrm{TTR} = 0$ has twice as much probability than the others). This also explains the mismatch observed in Figure 2. In all cases, the general model acts as an upper bound; which means that could be expect better results when real rendezvous sequences, like SSB, are used.

In the third experiment, the ETTR_M is evaluated by increasing M for different values of ρ. The results are shown in Figure 4. In all cases, ETTR_M increases with the increase of M; showing a fast growth as the PUs activity increases.

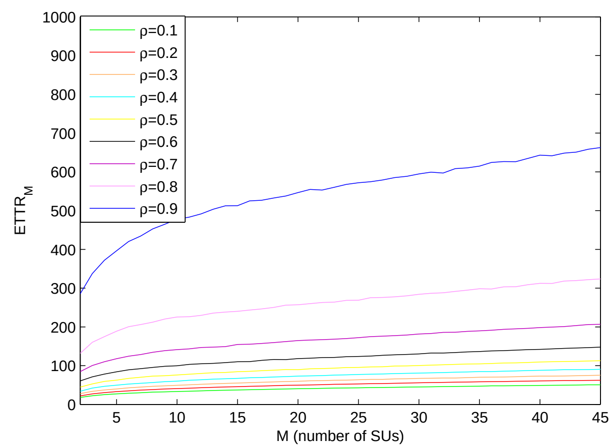

Fig. 4: ETTR for multi-hop ad-hoc network with different values of ρ, using the SSB algorithm with two radios varying the number of SUs.

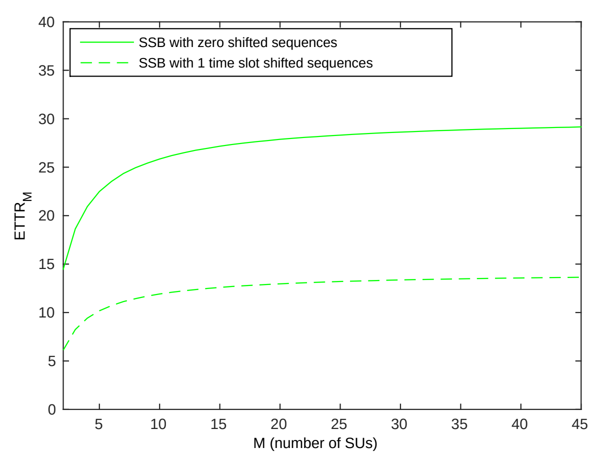

Fig. 5: ETTR for multi-hop ad-hoc network using the SSB algorithm with two radios varying the number of SUs.

In the above experiments the two radios were configured in each SU with zero shift between the sequences. However, as suggested at the bottom of Section II, some gain could be expected if the sequences are shifted to each other. For the SSB, several trials shifting were performed that changed the CH sequences. The best results were obtained for a shift between sequences equal to one time slot. In Figure 5, it can be observed that, for the shifted sequences, ETTR$_M$ decreased to approximately half of value obtained in the previous experiments. This is due to a reduction of the MTTR$_2$.

TABLE I: MTTR results for the SSB algorithm with two radios.

N	MTTR$_M$ (zero shift)	MTTR$_M$ (1 time slot shift)
2	2	0
3	4	1
4	6	2
5	8	3
6	10	4
7	12	5
8	14	6
9	16	7
10	18	8
11	20	9
12	22	10
13	24	11
14	26	12
15	28	13
16	30	14
17	32	15
18	34	16
19	36	17
20	38	18
...	...	...
30	58	28
...	...	...
40	78	38
...	...	...
100	198	98

Table I shows the MTTR$_M$ for different values of N (note that the synchronization strategy guarantees the same MTTR$_M$ irrespective of M). For zero shifted sequences, since the two radios follow exactly the same jump pattern, the behavior in terms of MTTR is identical to that observed with just one radio. As proved in [10], in this case, the MTTR$_M$ can be expressed as $2N-2$. A simple inspection of the values in Table I showed that for one time slot shift between the sequences, the MTTR$_M$ can be computed as $N-2$. That is approximately half of the value obtained for the zero shifted case, which explains the results observed in Figure 5.

V. Conclusions

This paper has described and analyzed a network synchronization strategy for rendezvous in multi-hop ad-hoc networks. A mathematical framework for the numerical computation of the ETTR$_M$ considering the presence of PUs was presented. The estimated ETTR$_M$ was compared with those obtained by simulation using a representative blind rendezvous CH sequence. The results showed that the analytic expressions provide good estimates of the actual ETTR. Moreover, the ETTR achieved by the proposed synchronization strategy has proven to be bounded by MTTR of a pair-wise rendezvous process irrespective of the number of SUs and the network topology. These results suggest that a proper combination of the CH sequences and the proposed synchronization strategy can bring a significant reduction of time required to establish a control channel and set up an ad-hoc network in large and dense deployment scenarios like those emerging in IoT applications.

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